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System-of-Systems Inspired Aircraft Sizing and Airline Resource Allocation via Decomposition

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The phrase "system-of-systems" describes a large system of multiple systems, each capable of independent operation, which have been brought together to provide capabilities beyond those of each individual constituent system. Formulating and solving a system-of-systems design problem has become increasingly important in the aerospace and defense industries as customers have begun to ask contractors for broad capabilities and solutions rather than for specific individual systems. Part of a system-of-systems design problem is determining the appropriate mix of both existing and yet-to-be-designed systems. Whereas determining an appropriate mix of existing systems falls into the category of resource allocation, including features of a yet-to-be-designed system complicates the problem by requiring the allocation of a variable resource. In this paper, an airline wishing to investigate how a new, yet-to-be-designed aircraft will impact the fleet operating costs provides a simple example of this type of problem. The resulting statement is a mixed-integer, nonlinear programming problem. Both a traditional approach and a new decomposition approach were used to solve the problem. The traditional approach is a mixed-integer nonlinear branch-and-bound. The decomposition approach applied to the problem is analogous to those of multidisciplinary optimization, in which there is an allocation domain and an aircraft sizing domain. When the problem size increases to the point where the traditional mixed-integer, nonlinear programming approaches cannot obtain a solution, the decomposition approach can find solutions for these larger problems. The multidisciplinary design optimization-motivated decomposition approach appears to have promise for the allocation of variable resources challenge presented by many system-of-systems design problems.

Nomenclature

cap_i	=	passenger capacity of existing aircraft type i
c_{ij}	=	cost coefficient of aircraft type i on route j
d_j	=	passenger demand on route j
s_i	=	number of trips available on aircraft type i
s_{TO}	=	takeoff field length
T/W	=	thrust-to-weight ratio
W/S	=	wing loading
x_{ij}	=	number of trips on aircraft type i on route j
y_X	=	passenger capacity of yet-to-be-designed aircraft
$()_i$	=	aircraft type, e.g., A, B, X
$()_j$	=	route number, e.g., 1, 2, 3

I. Introduction

THE term system-of-systems (SoS) is often used; however, there is not yet a single, widely accepted and consistently applied definition of a system-of-systems. Sage and Cuppan [1] claim a system-of-systems exists when there is a majority of five characteristics in the overall system: operational and managerial independence, geographic distribution, emergent behavior, and evolutionary development. Maier [2] suggests that a "systems-of-systems should be distinguished from large monolithic systems by the independence of their components, their evolutionary nature, emergent behaviors, and a geographic extent that limits the interaction of their components to information exchange." Keating et al. [3] describes their view of systems-of-systems as metasystems that "are themselves composed of multiple autonomous embedded complex systems that can be diverse in technology, context, operation, geography, and conceptual frame."

Although a single, widely accepted definition has not yet emerged, these descriptions of systems-of-systems share several common themes. For the work presented in this paper, SoS describes a concept in which many independent, self-contained systems are brought together to provide a capability or set of capabilities. The synthesis of these systems-of-systems presents different challenges than those presented by the complex, but single, large-scale system (e.g., a single aircraft, ground vehicle, etc.) design addressed by current engineering methods.

The importance of studying systems-of-systems is evident in the way customers (most notably the Department of Defense) of large engineering companies have changed their approach in acquiring new assets. Instead of asking for a single aircraft, vehicle, or other system to meet a set of performance requirements, the customers are asking for more encompassing solutions or capabilities. For instance, the recent U.S. Coast Guard Integrated Deepwater System program asked contractors to address the issue "that the Coast Guard needs to replace its aging assets while improving their technological capabilities [4]." Rather than specifying the mix of assets or assigning a set of requirements for the design of new assets, the Coast Guard described its offshore missions and gave contractors free rein to design a system that will meet its goals. A satisfactory solution requires an appropriate mixture of existing and new independently operating systems.

II. Problem Discussion

A. One Aspect of SoS Design as an Optimization Problem

The additional complexity of system-of-systems design arises, in part, because each of the independent systems could perform several tasks that would contribute to meeting the overall global need. In some cases, different systems could perform the same basic tasks (e.g., different aircraft could perform several parts of the Coast Guard's defined missions for the Deepwater program), and so assigning the appropriate system to the task to best meet the global need becomes more challenging. A new system that will operate within the system-of-systems must be designed with the goals of

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improving the system-of-systems' global performance; this is different from the traditional practice of working to improve the individual system's performance [5].

The hypothesis forwarded here is that decomposition approaches like those used for multidisciplinary design optimization (MDO), which are quite successful for the design of many engineering systems, may be combined with linear and integer programming techniques to allow for the generation of new system designs in a larger SoS context. In effect, the system-of-systems problem becomes an allocation problem in which some of the resources have variable properties. This paper improves upon and expands preliminary investigations undertaken by the authors [6]. The main contribution of this study is the decomposition of a large mixed-integer nonlinear programming (MINLP) problem into a small nonlinear programming (NLP) problem and moderate-size integer programming (IP) problem, here demonstrated for an airline that wishes to allocate a resource with variable characteristics (i.e., a yet-to-be-designed aircraft) along with existing resources. The decomposition approach/methodology presented in this study may be applicable to other "allocation of variable resources" problems.

The simple example used for these initial investigations makes use of several simplifications. The problems presented next demonstrate the methodology for solving an aircraft sizing problem in the context of system-of-systems. These problems are static; in practice, systems-of-systems will evolve over time with the change of the global needs and with the retirement of old constituent systems and the introduction of new systems. Also, the problems are deterministic; systems-of-systems will perform in uncertain environments and may be more appropriately modeled using probabilistic approaches. The trip demand of an airline, for instance, would be a good candidate for probabilistic treatment. Future investigations could address the uncertainty sources in the problem while using a decomposition approach like that presented in this study. Additionally, the problems assume that the airline has a given, fixed route structure; introducing a new aircraft could allow an airline to operate on new routes. Granted, the simplifications reduce the fidelity of the problem addressed, but these allow initial attempts at formulating and solving allocation of variable resources problems. Further specific assumptions are made in the problem description.

Beyond [5,6], there appear to be few similar investigations with a "design in an SoS context" motivation, to the authors' best knowledge. Recently, Taylor and de Weck [7] considered the design of an aircraft transportation network for overnight package delivery, in which the SoS problem addresses network design, aircraft design, and operations of an aircraft-based overnight package delivery entity via an "integrated" optimization approach, rather than a decomposition approach. References [8,9] describe early investigations that address simultaneous airline allocation and aircraft sizing over multiple time stages. Reference [8] follows a decomposition approach like that described in this paper; [9] uses an approach based upon analytic target cascading.

B. Example Problem

Here, an airline provides several features of a system-of-systems. In this scenario, several different aircraft (each an independent system) must be coordinated to provide a global capability; here the capability is transporting passengers. A "best" scenario results from determining how the aircraft collaborate to best provide the airline's capability. Given the various definitions of systems-of-systems, some may argue that an airline is not an SoS, because the airline makes corporate decisions about the routes on which each aircraft operates, hence, this limits "managerial independence." However, each aircraft is capable of operations that are independent of other aircraft owned by the airline; this could be considered an SoS with a highly centralized structure. Also, each aircraft maintains geographic distribution from the other aircraft. As a result, the airline meets several of the various SoS criterion.

As presented here, the airline seeks to minimize the cost of operating the fleet while meeting passenger demand. If the airline considers only the aircraft in its current fleet, integer design variables represent the numbers of each available aircraft type operated on each route. Existing resource allocation techniques can solve this problem. The complexity increases when the airline considers a yet-to-be-designed aircraft as an option and design variables describing the new aircraft are included in the problem statement. The performance coefficients of the yet-to-be-designed aircraft are nonlinear functions of the aircraft's design variables, and so the combined aircraft sizing/fleet allocation problem becomes mixed-integer, nonlinear.

1. *Traditional Aircraft Allocation Problem*

Dantzig's linear programming textbook [10] presents one of the earliest discussions of airline allocation as a basic operations research problem, and numerous examples of work to advance aircraft allocation and scheduling exist [11–15]. A homework problem given in Bazaraa and Jarvis [16] provides the first example aircraft allocation problem used here. This is a simple and common operations research problem, however, extensions of this problem appear to provide a basis for SoS investigations, much like the three-bar truss problem served for structural optimization.

In this problem, an airline wishes to assign two types of aircraft to three routes to minimize cost. Each aircraft owned by the airline company can make no more than two trips per day. There are three available aircraft of type A and four available aircraft of type B. Each A aircraft has a capacity of 140 passengers; each B aircraft, 100. The passenger demands for each of the three routes serviced by the airline are 300, 700, and 220. No distinction is given between economy and first class passenger demand. Based upon the cost coefficients in Bazaraa and Jarvis' example, the routes are of decreasing length (route 1 has the highest cost coefficient, route 3 has the lowest). Here, route 1 is assumed to be 2000 n miles in range, route 2 is 1500 n miles, and route 3 is 1000 n miles. Considering a hub-and-spoke system, each route in this problem links the hub airport and an outlying airport, as Fig. 1 illustrates.

An aircraft trip is a round trip (one leg outbound and a return leg to the hub). Following the one-way demand idea in Dantzig's aircraft allocation problem [10], a given route's demand is the larger of the number of passengers traveling from an outlying city to the hub or the number of passengers traveling from the hub to the outlying city in one day. With this assumption, if 300 passengers want to travel from city 1 to the hub each day, the demand on the leg from the hub back to city 1 is less than or equal to 300 passengers. In hub-and-spoke operations, some of the airline's 300 passengers traveling from city 1 to the hub will continue on to city 2 or 3. Because this example problem does not consider scheduling the times of flights during the day and uses a fixed route structure, the one-way demand values account for all passenger origin/destination pairs. Those passengers who are traveling on to city 2 after arriving at the hub from city 1, for instance, are included in the 700 passenger demand on route 2.

For the investigations discussed here, the objective is to minimize the airline's total direct operating cost (DOC) for one day of operations. Direct operating costs for each aircraft normally involve fuel consumption, crew costs, maintenance, depreciation, and insurance [17]. Predictors for DOC also include contributions of trip time, empty weight, landing weight, and takeoff gross weight. To

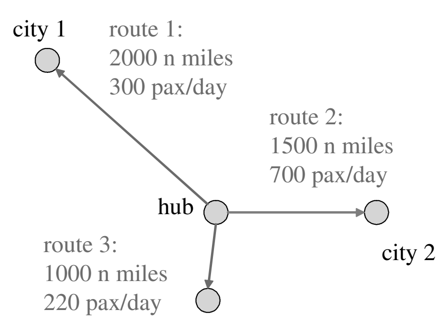

Fig. 1 Three-route airline allocation problem.

Table 1 Aircraft characteristics obtained from [17]

Parameter	Value	Parameter	Value	Parameter	Value
Cockpit length, in.	130	horizontal tail c/4 sweep, deg	30	vertical tail c/4 sweep, deg	35
Lavatory space, in.	40	horizontal tail taper ratio	0.50	vertical tail taper ratio	0.50
Closet space, in.	50	horizontal tail t/c ratio	0.10	vertical tail t/c ratio	0.12
Nose fineness ratio	0.25	horizontal tail aspect ratio	5.15	vertical tail aspect ratio	1.63
Tail fineness ratio	1.50	horizontal tail volume coefficient	1.20	vertical tail volume coefficient	0.12

compute the DOC of aircraft type A and type B on each route, size and performance characteristics are needed for each aircraft. An aircraft sizing and DOC predicting tool was employed for this task. For the first studies, this consisted of using MATLAB scripts to develop a weight prediction and sizing code based on approaches presented in Raymer [17], Shevell [18], and Anderson [19], and the cost prediction equations presented in [20]. Mane [21] provides additional description of the weight and DOC predictions in this sizing code.

The passenger capacities of aircraft type A (140 passengers) and B (100 passengers) closely resemble the capacities of the Boeing 737-400 (146 passengers) and 737-500 (108 passengers), and so parameters for these 737 aircraft models from [22] provided input to the MATLAB code for sizing and DOC prediction. The information available from [22] does not include all input values required by the aircraft sizing code. For those input values not included in [22], empirical values for commercial transport aircraft presented in Raymer [17] were used. Table 1 presents values of the most important parameters from [17]; a detailed list also appears in [21].

Using empty weights calculated during the design mission and assuming a full passenger load on all trips on each of the routes, takeoff weights, fuel weights, and DOC values for the economic missions can be computed. The aircraft sizing and economic missions all follow the basic mission profile given in Fig. 2.

For the economic missions, the aircraft carries fuel to complete the mission assuming a constant speed cruise-climb of Mach 0.765 starting at an altitude of 30,000 ft and including the reserve segment, but the mission fuel consumed does not include the reserve loiter for cost calculations. The DOC cost coefficients for each route of the existing fleet as computed by the MATLAB sizing tool are summarized in Table 2. The DOC-per-trip values reflect a round trip (hub–city–hub).

With the cost coefficients determined, the problem follows a typical integer programming formulation. The design variables x_{ij} represent the number of trips of aircraft type i. The objective is to minimize the DOC for the fleet over one day of operations as represented by Eq. (1). Because no aircraft can make more than two trips per day and the airline only owns three type A aircraft, no more than six trips can be made by A aircraft; similarly, no more than eight trips can be made by the airline's four type B aircraft. Equation (2) represents this constraint. The airline must meet the one-way passenger demand on all three routes, represented by Eq. (3). With this information, Eqs. (1–5) form the integer programming problem statement.

Minimize

$$f = c_{A1}x_{A1} + c_{A2}x_{A2} + c_{A3}x_{A3} + c_{B1}x_{B1} + c_{B2}x_{B2} + c_{B3}x_{B3} \quad (1)$$

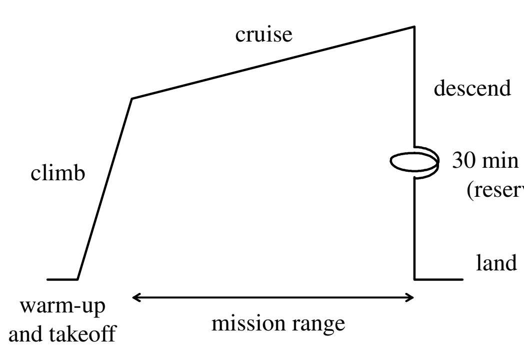

Fig. 2 Basic mission profile.

subject to

$$\sum_{j=1}^{3} x_{ij} \le s_i \quad (2)$$

$$\sum_{i=A}^{B} \text{cap}_i x_{ij} \ge d_j \quad (3)$$

$$x_{ij} \ge 0 \quad (4)$$

$$x_{ij} \text{integer} \quad (5)$$

2. Airline Allocation Including Design of a New Aircraft

In this scenario, the airline company wishes to assign aircraft to its three routes, and it currently has three aircraft of type A and four aircraft of type B in its fleet. However, the company is now considering how the yet-to-be-designed aircraft type X would impact its three-route operations. The passenger demand is unchanged from the previous problem statement. As before, each aircraft can make no more than two trips per day. In this formulation, the airline plans to acquire up to three of the new type X aircraft.

The objective remains to minimize the DOC for one day of operations by the airline's fleet. The cost coefficients for aircraft types A and B are the same as in Table 2. The cost coefficients and capacity of aircraft type X are a function of the variables describing this aircraft, so the problem now involves the allocation of a variable resource. The number of passengers, wing loading, aspect ratio (AR), and thrust-to-weight ratio for the type X aircraft are included as design variables.

Minimizing DOC, while matching the formulation of the problem from Bazaraa and Jarvis [16], is an incomplete representation of the problem, because an airline's goal is to maximize profit while meeting passenger demand. Fleet DOC does not directly include the initial acquisition cost associated with procuring a new aircraft or other fixed costs associated with certain routes or aircraft types, and it ignores indirect operating costs (IOC). Estimation of IOC includes detailed airport and maintenance models which vary from operator to operator. Estimating revenue requires pricing strategies which also vary among airlines. Minimizing DOC is a surrogate for maximizing profit by omitting revenue and IOC estimates. This omission, however, has no effect on the algorithmic insights gained from this study or the validity of the methodology presented here. Because the definition of DOC is consistent throughout the study, the comparisons of the solution methodologies are valid.

The same MATLAB-based aircraft sizing tool used to predict cost coefficients of aircraft types A and B also predicts the cost coefficients of the type X aircraft. It is assumed that the type X aircraft

Table 2 Predicted cost coefficients (DOC per trip) for aircraft type A and B

Aircraft type	Route 1 DOC, \$	Route 2 DOC, \$	Route 3 DOC, \$
A	33,030	27,718	22,450
B	28,732	24,070	19,431

is a twin-engine, narrow-body (single-aisle) passenger transport aircraft. The type X aircraft has two flight crew members. The number of passengers is a variable. The number of first class passengers is 10% of the total capacity, and the remaining passengers are economy class. The passenger capacity of the aircraft determines the number of cabin crew.

For aircraft X, the design mission matches the airline's longest route (here, 2000 n miles), and the DOC per trip on this longest route is calculated during sizing. Using the empty weight computed for the design mission and assuming a full crew and passenger load, the sizing code also estimates the DOC associated with trips on the remaining routes.

The cost coefficients for aircraft types A and B, c_{Aj} and c_{Bj}, are constants, but the cost coefficients for aircraft type X are functions of the variables y_X, $(W/S)_X$, $(T/W)_X$, and AR_X. To ensure that aircraft X can operate at all airports used by the airline, a constraint limits takeoff field length to no greater than 8990 ft, based upon aircraft A's predicted field length of 8990 ft, the longest distance predicted for either aircraft type A or B. Bounds ensure that the aircraft X design variables stay within ranges deemed reasonable for twin-engine, narrow-body transport aircraft. Equations (6–15) represent this new problem formulation.

Minimize

$$\begin{aligned} f = {} & c_{A1}x_{A1} + c_{A2tA2} + c_{A3}x_{A3} + c_{B1}x_{B1} + c_{B2}x_{B2} + c_{B3}x_{B3} \\ & + c_{X1}[y_X, (W/S)_X, (T/W)_X, \mathrm{AR}_X]x_{X1} \\ & + c_{X2}[y_X, (W/S)_X, (T/W)_X, \mathrm{AR}_X]x_{X2} \\ & + c_{X3}[y_X, (W/S)_X, (T/W)_X, \mathrm{AR}_X]x_{X3} \end{aligned} \tag{6}$$

subject to

$$\sum_{j=1}^{3} x_{ij} \le s_i \tag{7}$$

$$\sum_{i=A}^{B} \mathrm{cap}_i x_{ij} + y_X x_{Xj} \ge d_j \tag{8}$$

$$s_{\mathrm{TO}}[y_X, (W/S)_X, (T/W)_X, \mathrm{AR}_X] \le 8990 \tag{9}$$

$$x_{ij} \ge 0 \tag{10}$$

$$80 \le y_X \le 250 \tag{11}$$

$$95 \le (W/S)_X \le 150 \tag{12}$$

$$6 \le \mathrm{AR}_X \le 9.5 \tag{13}$$

$$0.3 \le (T/W)_X \le 0.4 \tag{14}$$

$$x_{ij}, y_X \text{integer} \tag{15}$$

This appears to be a simple problem with 13 variables and seven inequality constraints, but the $c_{Xj}x_{Xj}$ terms in the objective function and $y_X x_{Xj}$ in the demand constraints make this a mixed-integer nonlinear programming problem. MINLP problems are typically difficult to solve.

Table 3 Solution to baseline airline allocation problem

Variable, constraint, or objective	Value
x_{A1} (trips by aircraft A on route 1)	0
x_{A2} (trips by aircraft A on route 2)	5
x_{A3} (trips by aircraft A on route 3)	1
x_{B1} (trips by aircraft B on route 1)	3
x_{B2} (trips by aircraft B on route 2)	0
x_{B3} (trips by aircraft B on route 3)	1
Total passenger capacity on route 1	300
Total passenger capacity on route 2	700
Total passenger capacity on route 3	240
Fleet DOC for one day	$266,667

III. Methods and Approach

The first step is to pose and solve the traditional aircraft allocation IP problem to identify a feasible baseline allocation. Then, strategies based upon concepts from multidisciplinary design optimization allow a solution of the MINLP problem which includes allocation of a variable resource.

A. Traditional Airline Allocation Problem Solution

With the traditional allocation statement presented in Eqs. (1–5), integer programming methods can solve the problem. Here, the General Algebraic Modeling System (GAMS) software [23], accessed through a MATLAB interface [24], provided a solution using the IP solver CPLEX [23]. Table 3 summarizes this baseline airline allocation solution. This solution's capacity exactly meets the demand constraint on routes 1 and 2 and slightly exceeds the demand on route 3. The solution exactly meets the trip count limit for aircraft type A and is well below the trip count limit for aircraft type B.

Because the problem is small in size (few variables and constraints), the solution is obtained by CPLEX in seconds. These results provide a baseline solution for comparison with studies of the allocation of a variable resource version of the problem.

B. Airline Allocation Including Sizing of a New Aircraft

Including the sizing of a new aircraft as part of the problem statement results in a mixed-integer, nonlinear problem. However, techniques developed for multidisciplinary design optimization appear promising as solution methods for this particular construct. MDO approaches use the disciplinary boundaries of coupled, multidisciplinary problems to define the problem [25]. Using this concept, the airline allocation problem including the design of a new aircraft has two "disciplines": resource allocation and aircraft sizing.

The standard or multidisciplinary feasible MDO approach combines the two disciplines into a single analysis. This follows the problem statements presented in Eqs. (6–15); solving this problem with an appropriate MINLP method, like nonlinear branch-and-bound, requires significant computational effort.

MDO decomposition strategies separate the discipline-specific analyses of problems to reduce computational effort associated with integrating two separate analysis codes for an iterative analysis. MDO decomposition strategies generally reside in two broad categories: 1) all-at-once or optimizer-based, and 2) discipline-feasible constraint or subspace optimizations. The discipline-feasible approach appears promising as a guide for the allocation of variable resources problem because aircraft sizing and resource allocation appear to be two disciplines present in this problem. Decomposing the problem into these two subproblems and then solving each subproblem via discipline-specific tools (nonlinear programming for the aircraft sizing and integer programming for the fleet allocation) reduces overall computational effort. The optimizer-based decomposition would require one optimization problem with all variables included, so that the optimizer must still address a single, large MINLP problem as it would in the standard approach. This would offer little or no computational advantage.

Table 4 MATLAB branch-and-bound solutions to three-route allocation problem

Variable, constraint, objective, or computational effort	Warm start $X = A$	Warm start $X = B$	Warm start $X = 757$	Cold start $X = A$	Cold start $X = B$	Cold start $X = 757$	Cold start $X =$ lower bounds
x_{A1}	0	No solution obtained	0	0	0	0	0
x_{A2}	0		0	0	0	0	0
x_{A3}	0		0	0	0	0	0
x_{B1}	1		1	1	1	1	1
x_{B2}	0		0	0	2	0	0
x_{B3}	0		0	0	0	0	0
x_{X1}	1		1	1	1	1	1
x_{X2}	3		3	3	2	3	3
x_{X3}	1		1	1	1	1	1
y_X	234		234	234	250	234	234
$(W/S)_X$, lb/ft^2	126.55		126.55	126.55	126.55	126.55	126.55
AR_X	6.00		6.00	6.00	6.00	6.00	6.00
$(T/W)_X$	0.30		0.30	0.30	0.30	0.30	0.30
Capacity on route 1	334		334	334	350	334	334
Capacity on route 2	702		702	702	700	702	702
Capacity on route 3	234		234	234	250	234	234
$(s_{\mathrm{TO}})_X$, ft	8990		8990	8990	8900	8990	8990
Fleet DOC + I/day, $	229,497		229,497	229,497	247,694	229,497	229,497
Function evaluations	30,602		11,171	11,380	23,569	10,850	41,539

1. Standard Approach

Following the standard approach, the problem presented in Eqs. (6–15) is solved as an MINLP problem using the MATLAB-based branch-and-bound routine, BNB20 [26]. This routine uses the sequential quadratic programming algorithm available from the Mathworks' MATLAB optimization toolbox [27] to perform the constrained nonlinear search. The objective function needed by the algorithm uses the MATLAB aircraft sizing code, described earlier, to obtain the direct operating cost coefficients for the new aircraft X. These coefficients contribute to the total DOC for the entire fleet and are functions of the aircraft X design variables.

Nonlinear branch-and-bound methods are susceptible to finding the equivalent of local minima resulting from the path options chosen during the search. To address this, the solver began its search from several different starting values for the aircraft X design variables as well as the airline fleet allocation variables. Enumerating all possible solution combinations would be a way to verify the global optimality of the solution, however, this is computationally prohibitive even for the simple problem considered here. Table 4 presents the solution generated by BNB20 for each initial condition. "Warm start" indicates that the run began using the allocation of A and B aircraft in Table 3; "cold start" indicates that the run began with no aircraft allocated. The $X = A$, $X = B$, and $X = 757$ indicate that the run began using the wing loading, aspect ratio, thrust-to-weight ratio, and passenger capacity of aircraft A, B, and of the Boeing 757, respectively, as the initial description of the new aircraft. The last run used the lower bounds in Eqs. (11–14) as the initial description of aircraft X. Reference [21] describes these initial conditions in more detail.

Here, five of the seven runs obtain the allocation and aircraft X description that result in a $229,497 fleet DOC per day. This figure represents a reduction of 14% from the $266,667 value obtained by allocating only the existing aircraft A and B (Table 3). One of the runs did not reach its stopping criterion and one of the runs found a local-minima equivalent solution. The computational cost associated with the branch-and-bound algorithm is quite high. The longest run of the BNB20 algorithm that obtained a solution required 41,539 objective function evaluations. The elapsed time for these runs ranged from about 30 to about 85 min, when conducted on a shared processor machine with varying loads during different runs. However, if enumeration were considered, instead of branch-and-bound, calculating all combinations of the integer variables would require more than 10^9 function evaluations.

2. MDO Decomposition Approach

In a discipline-feasible constraint approach, the multidisciplinary problem decomposition results in a system-level problem with several subspace problems [28]. For the airline allocation subproblem, the objective is to minimize the fleet DOC for one day of the airline's operations while satisfying the demand and trip limit constraints. The only inputs that this level needs are the cost coefficients for each route for the new aircraft X and the passenger capacity. The aircraft sizing subproblem minimizes the DOC per trip for the new aircraft X for a given passenger capacity by changing the aircraft design variables AR_X, $(W/S)_X$, and $(T/W)_X$. The only constraint in this level is the takeoff field length.

This decomposition strategy presents the allocation subproblem as an integer programming problem and the aircraft sizing subproblem as a nonlinear programming problem. The variables describing the number of aircraft trips flown by each aircraft type on each route are discipline specific to the airline allocation analysis, whereas the variables describing the new aircraft's characteristics, AR_X, $(W/S)_X$, $(T/W)_X$, are discipline specific to the aircraft sizing analysis. The variable describing the passenger capacity of the new aircraft type X is multidisciplinary, because it is used in both the allocation analysis and the aircraft sizing analysis.

In the system-level or top-level problem, the objective is to minimize the DOC of the fleet for one day of operations. The only multidisciplinary variable is the passenger capacity and there are no constraints. This, in itself, is an unconstrained univariate integer problem that a modified golden section search [29] can solve by rounding each new variable value generated during the search to the nearest integer. In this modified search, the convergence criterion is when the two middle values in the golden section interval are adjacent integers. The initial upper and lower values provided to the golden section search are the bounds in Eq. (11).

The decision variable at the top level is the passenger capacity for the new aircraft X. As Fig. 3 shows, the aircraft sizing subproblem needs the passenger capacity as input. It optimizes the aircraft to minimize DOC on the design route using AR_X, $(W/S)_X$, and $(T/W)_X$ as design variables, while holding y_X constant at the value provided by the golden section search.

In the aircraft sizing subproblem, the objective is to minimize the DOC for the new aircraft on the design route. This subproblem uses the local decision variables AR_X, $(W/S)_X$, and $(T/W)_X$, and the passenger capacity provided by the system level to find a description of aircraft type X that minimizes the DOC for the longest route (design mission), 2000 n miles, that the aircraft has to travel. In this formulation, the new aircraft can fly all of the airline's routes, however, in a more general model, design range of the new aircraft could be a top-level design variable. One of the implications of such a formulation would be that the top-level problem would have one integer variable (capacity) and one continuous variable (range). This would require a mixed-integer, nonlinear programming solver and would incur more computational expense, however, this MINLP

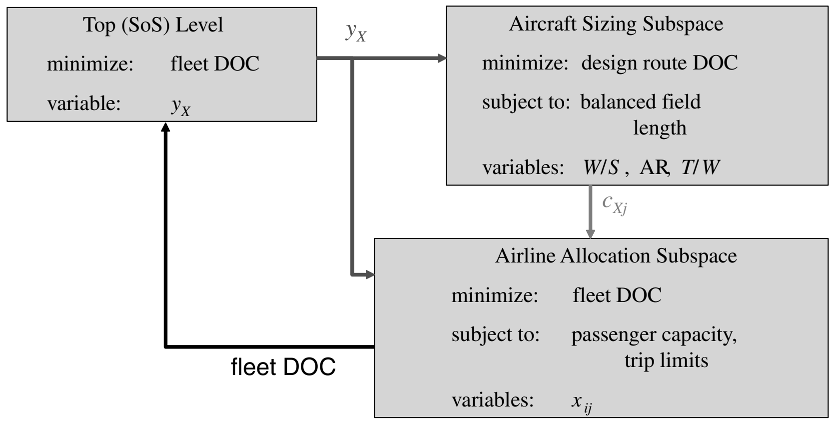

Fig. 3 Sequential decomposition schematic.

problem is significantly smaller than the standard approach's MINLP problem. The sequential decomposition presented in Fig. 3 is likely the correct formulation of the problem including range as a variable. The subproblems remain NLP for the aircraft sizing and IP for the resource allocation.

The sizing problem also enforces the takeoff field length constraint, because this is a function of only the aircraft sizing domain variables. The aircraft sizing subproblem is a nonlinear programming problem as described by Eqs. (16–20):

Minimize

$$f = (\text{DOC}_{2000\ \text{n miles}})_X \tag{16}$$

subject to

$$s_{\text{TO}}[y_X, (W/S)_X, (T/W)_X, \text{AR}_X] \leq 8990 \tag{17}$$

$$95 \leq (W/S)_X \leq 150 \tag{18}$$

$$6 \leq \text{AR}_X \leq 9.5 \tag{19}$$

$$0.3 \leq (T/W)_X \leq 0.4 \tag{20}$$

The sizing problem generates the cost coefficients for the new type X aircraft. These cost coefficients, c_{X1}, c_{X2}, c_{X3}, and the passenger capacity y_X, become inputs to the aircraft allocation subproblem and have constant values in the allocation subproblem. The result is an integer programming formulation with constant cost coefficients and linear constraints. The local decision variables are the number of trips that aircraft type i makes on route j. Equations (21–25) describe this allocation subproblem:

Minimize

$$f = c_{A1}x_{A1} + c_{A2}x_{A2} + c_{A3}x_{A3} + c_{B1}x_{B1} + c_{B2}x_{B2} + c_{B3}x_{B3} + c_{X1}x_{X1} + c_{X2}x_{X2} + c_{X3}x_{X3} \tag{21}$$

subject to

$$\sum_{j=1}^{3} x_{ij} \leq s_i \tag{22}$$

$$\sum_{i=A}^{B} \text{cap}_i x_{ij} + y_X x_{Xj} \geq d_j \tag{23}$$

$$x_{ij} \geq 0 \tag{24}$$

$$x_{ij} \text{integer} \tag{25}$$

The local constraints limit the number of trips per day for each aircraft and provide capacity to meet passenger demand. The solution of the traditional allocation problem using only aircraft A and B is a known, feasible allocation for this problem and is a possible solution when aircraft X is introduced.

A series of MATLAB scripts coordinates the top system-level problem and the two-domain-specific sublevel problems. An interface between MATLAB and the GAMS software [24] allows variables generated by the aircraft sizing code to become input for GAMS. The modified golden section search encoded in MATLAB solves the system-level problem, the GAMS integer programming algorithm solves the allocation problem, and MATLAB solves the aircraft sizing problem.

To obtain the optimal solution in Table 5, the system-level problem makes ten iterations; the aircraft sizing domain problem and the allocation domain problem are each called 10 times. However, the GAMS software makes use of good algorithms for solving integer programming problems, and the entire golden section search concludes in about 8.3 min. It took up to 85 min to solve the problem in a monolithic representation, and multiple initial conditions occasionally resulted in a suboptimal solution or no solution at all.

The optimal passenger capacity of aircraft type X is 234; this indicates that a fairly large, single-aisle aircraft would reduce the DOC of the airline presented here. Because passenger demand and trip limit are the only constraints, the optimal aircraft should carry as many passengers as possible to meet the passenger demand with the least possible cost. Greater carrying capacity means smaller DOC per seat-mile. Recall that this problem does not capture many details that are also part of a real fleet assignment problem. For instance, the constraints do not reflect time scheduling of flights; as a result, the airline has only one flight on route 3 per day.

Table 5 Solutions to sequential decomposition

Variable, constraint, or objective	Value
x_{A1}	0
x_{A2}	0
x_{A3}	0
x_{B1}	1
x_{B2}	0
x_{B3}	0
x_{X1}	1
x_{X2}	3
x_{X3}	1
y_X	234
$(W/S)_X$, lb/ft^2	126.55
AR_X	6.00
$(T/W)_X$	0.30
Capacity on route 1	334
Capacity on route 2	702
Capacity on route 3	234
$(s_{\text{TO}})_X$, ft	8990
Fleet DOC + I/day, $	229,497

Table 6 DOC per seat-mile for each route in the three-route problem, assuming full passenger load

	Route 1, \$/seat-n mile	Route 2, \$/seat-n mile	Route 3, \$/seat-n mile
Aircraft type A	0.059	0.066	0.080
Aircraft type B	0.072	0.080	0.097
Aircraft type X	0.051	0.057	0.070

The allocation in Table 5 indicates the acquisition of three new type X aircraft and, with the new aircraft, the airline now needs only one type A and one type B aircraft to meet passenger demand. The DOC objective function does not capture expenditures associated with parking, retiring, or selling an aircraft; decisions regarding the useful life of an aircraft vary significantly from operator to operator. The cost prediction model does include depreciation and interest associated with acquiring a new aircraft, but does not include additional fixed costs associated with acquiring a new aircraft (e.g., new maintenance equipment, training). Incorporating these costs into the allocation domain problem would require additional terms in the objective function, but the problem would remain IP and the decomposition strategy would still work. With the aforementioned additional costs included, the number of new aircraft acquired might be reduced; using the decomposed problem formulation would facilitate this study.

Also, the allocation uses more new aircraft type X than either aircraft A or B. Table 6 presents DOC per seat-mile for each aircraft for the three routes' round trips. A primary reason for the lower cost of aircraft type X is that its design range (2000 n miles) exactly matches the existing route structure, whereas the design ranges of aircraft A and B (2080 and 2400 n miles, respectively) exceed the maximum distance route. Carried to an extreme, a new aircraft sized for each route may further reduce fleet cost. However, the acquisition of multiple new aircraft types simultaneously is unlikely, and the minimization of fleet DOC as a surrogate for maximizing profit becomes less believable because the fixed costs associated with introducing multiple new aircraft types would become significant.

This sequential decomposition approach also proves to be independent of initial conditions when solving the problem presented here. The concept of initial conditions changes at the system level. Golden section uses lower and upper bound values for the design variable (y_X^L and y_X^U) to start the search. The aircraft sizing subproblem (an NLP problem) was tested by imposing a series of initial conditions for the values of AR_X, $(W/S)_X$, and $(T/W)_X$. This did not affect the outcome of the optimization, suggesting that there is one constrained minimum for this subproblem. Results of the resource allocation subproblem optimization (an IP problem) were consistent and the computational time was insensitive to a series of initial conditions.

For this example problem, and the larger problems described next, the two-domain decomposition strategy is appropriate. The decomposition allows an increase in the complexity of the both the aircraft sizing subproblem and the resource allocation subproblem (e.g., more variables and constraints in either the allocation or aircraft sizing subproblems, higher-fidelity aircraft sizing analysis). Additional features, perhaps route network design (i.e., determining which city pairs are connected by direct flights) or crew scheduling, might introduce other domains into the system-of-systems inspired problem for which a multidomain decomposition is more appropriate. The methods employed in [30] to investigate the optimality of hub-and-spoke and alternative route networks may provide such an additional domain for route network design.

IV. Increase in Problem Size

To this point, the investigated problem provides a simplistic example of an airline's operations. Subramanian et al. [15] reports that several years ago Delta Airlines periodically scheduled approximately 2500 flights per day using about 400 aircraft of 10 different aircraft types. Feasible fleet assignment problems can have as many as 75,000 binary variables, 1000 integer variables, and more than 50,000 constraints. Investigating the ability of the preceding decomposition approach to solve problems of increasing size and complexity is appropriate.

A. Impacts of Increasing Problem Size

The directions of problem size growth available are many. To represent a realistic airline operation scenario in the problem statement could include scheduling, maintenance, routing, and other details. The complexity and size of the problem rise by increasing the number of aircraft owned by the airline, as well as increasing the number of routes that it serves. For example, the number of integer variables in the allocation problem is the product of the number of routes served and the number of aircraft types (existing types plus the new type X) used by the airline.

Although the decomposition approach provided solutions for the three-route problem with less computational effort than the standard approach, identifying the problem size limitation of the monolithic MINLP branch-and-bound further demonstrates that the decomposition allows solution of more reasonable sized problems. To find this limit, increments in the number of routes and existing aircraft increased the three-route problem size. First, adding one existing aircraft type so that the problem had three routes and four aircraft types increased the number of integer variables by three, from 10 to 13. Subsequent increments resulted in problems of four routes with four aircraft types (17 integer variables) and four routes with five aircraft types (21 integer variables). At the next increment, five routes with five aircraft types (26 integer variables), the branch-and-bound was unable to find a solution. Figure 4 displays the computational cost to solve the MINLP problem using the standard approach; the trend line shows the nearly exponential growth in cost.

Up to this point in the investigation, a series of MATLAB scripts developed by the authors provided the aircraft sizing predictions. This MATLAB-based aircraft sizing analysis allowed for easy implementation of the standard MINLP allocation of variable resource problem; the branch-and-bound algorithm employed to solve the MINLP problem was also MATLAB-based. The familiarity of the authors with MATLAB allowed for detailed scrutiny of the computational process and the results. Because the problem size quickly limits the standard approach to the airline allocation of a variable resource problem, the decomposition strategy is necessary for solving more realistic sized problems. With the decomposition, other sizing codes can replace the simple MATLAB scripts with no modification to the approach.

For the rest of the investigation, the Flight Optimization System (FLOPS) [31] provided size and performance predictions of the airline's existing aircraft and the yet-to-be-designed aircraft X; the

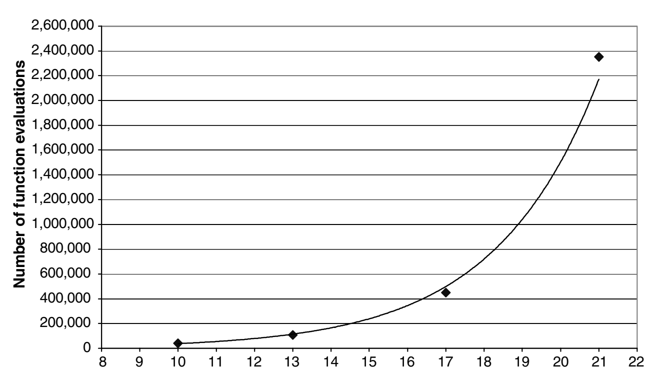

Fig. 4 Computational cost of branch-and-bound.

Table 7 Problem size comparison

Routes	Existing aircraft types	Integer variables	Continuous variables	Demand constraints in allocation problem	Trip limit constraints in allocation problem
3	2	10	3	3	3
11	6	78	3	11	7
31	7	249	3	31	8

cost-estimating routine [32] in FLOPS estimated the DOC of these aircraft. FLOPS is a multidisciplinary suite of computer routines for conceptual and preliminary aircraft design, as well as evaluation of advanced aircraft concepts [33], and is regularly used in aircraft sizing studies by NASA and the industry. This increases the detail and fidelity of the aircraft sizing predictions over the MATLAB-based scripts used in the studies described earlier.

The following sections present two larger problems in more detail. The first is an 11-route problem; the second, a 31-route problem. In these problems, the number of routes, different existing aircraft types, and available existing aircraft are all larger than the simple three-route example. The remaining simplifying assumptions remain unchanged. Table 7 compares the 11-route and 31-route problem sizes with the three-route problem.

B. Eleven-Route Problem

This study uses information about a major U.S.-based airline. Data obtained from the Bureau of Transportation Statistics (BTS) [34] describe this airline's operations from Boston, Massachusetts for the month of June 1999. These data include the regularly scheduled flights from Boston, the number of passengers on these flights, and the actual equipment used. Figure 5 displays the 11 regularly scheduled routes. Dividing the total number of passengers on each route for the month by the number of days in the month provided values for daily demand; these daily demand values appear in Table 8, which also lists the range between Boston and the 11 destination cities.

For the application presented here, the problem deviates from the actual airline. Boston becomes the hub airport, and the daily demand value on the outbound leg of a round trip becomes the one-way demand. Without considering the scheduling of flights during the day, the one-way demand values represent all of the passenger origin/destination pairs for the day. The airline only serves the 11 routes in Fig. 5. Keeping with the assumption that each aircraft can fly only two trips per day, the number of flights recorded in the BTS data allows calculation of the number of each aircraft "owned" by the airline in this problem. The airline owns two aircraft type A, A300-600 with a passenger capacity of 190; five aircraft type B, B757-200 with a passenger capacity of 180; one aircraft type C, B767-200 with a passenger capacity of 165 (configured in three-class layout); two aircraft type D, B737-800 with a passenger capacity of 146; seven aircraft type E, DC9 Super 80/MD81 with a passenger capacity of 139; and two aircraft type F, Fokker 100 with a passenger capacity of 93. The design range of each aircraft indicates whether or not a given aircraft is able to fly the required routes; for instance, the BOS–SEA distance exceeds the range of aircraft type F. The airline is considering the addition of a new yet-to-be-designed aircraft type to its fleet, and the goal is to minimize the cost of operating the fleet while meeting passenger demand.

To begin, each of the airline's existing aircraft's design range, along with other data describing their characteristics, obtained from *Jane's All The World's Aircraft* [22], becomes input to FLOPS for sizing, as in the previous three-route problem. The aircraft descriptions and the distances of each of the 11 routes provide the inputs that allow FLOPS to generate the performance and cost coefficients of each aircraft on each route.

For some input values not included in [22], FLOPS computes its own values based on equations describing commercial transport aircraft. An example commercial transport aircraft input file provided with the FLOPS code supplied the necessary settings required to compute values that were not available in [22]. Such examples include sweep angle of the horizontal and vertical tails as well as thickness-to-chord ratios for the tails.

Further, the design missions for these aircraft are not published in [22]; only general statements about maximum range appear. Because of this, FLOPS computed the design range of type A–F aircraft using fixed values of takeoff gross weight based on published values for the A300-600, B757-200, B767-200, B737-800, DC9 Super 80, and Fokker 100 aircraft by letting FLOPS find the optimal cruise Mach number for a given cruise altitude. The cost coefficients for each airplane on each route computed by FLOPS are used in the objective function of the allocation problem.

When an airline allocates its aircraft, it has to consider the timing of flights throughout the day, the location of its maintenance facilities, the maintenance schedule required for each aircraft, crew

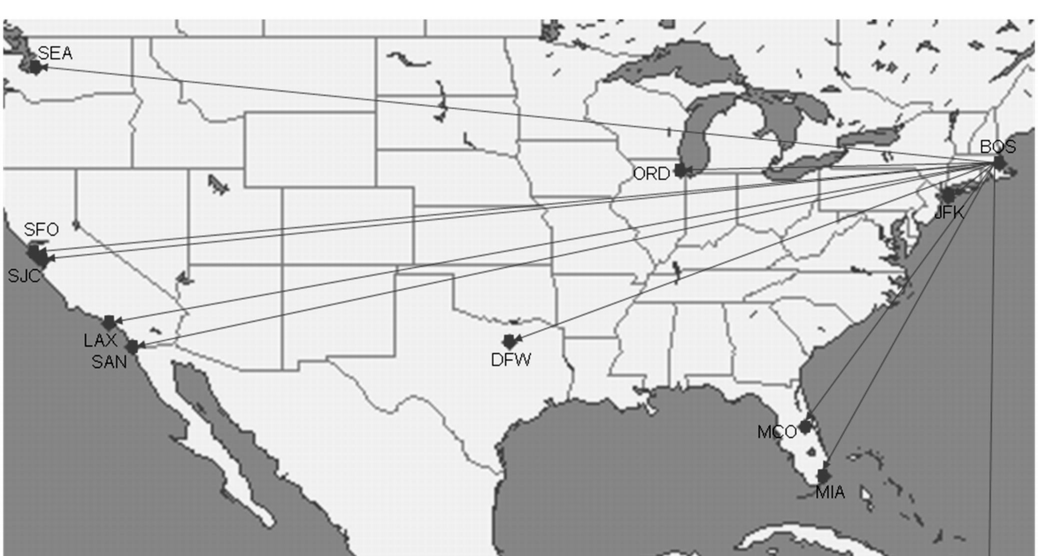

Fig. 5 Eleven-route allocation problem.

Table 8 Demand for each of 11 routes and distance from hub

Boston (BOS) to:	Route no.	Daily demand, pax	Distance, n mile
New York, NY (JFK)	1	41	162
Chicago, IL (ORD)	2	1009	753
Orlando, FL (MCO)	3	89	974
Miami, FL (MIA)	4	661	1094
Dallas, TX (DFW)	5	1041	1357
San Juan, Puerto Rico (SJU)	6	358	1455
Seattle, WA (SEA)	7	146	2169
San Diego, CA (SAN)	8	97	2249
Los Angeles, CA (LAX)	9	447	2269
San Jose, CA (SJC)	10	194	2337
San Francisco, CA (SFO)	11	263	2350

Table 9 Allocation of existing fleet for 11-route problem

Boston (BOS) to:	Route no.	a/c A A300-600	a/c B B757-200	a/c C B767-200	a/c D B737-800	a/c E DC9 S80	a/c F Fokker 100
New York, NY (JFK)	1	—	—	—	—	—	1
Chicago, IL (ORD)	2	—	2	—	—	4	1
Orlando, FL (MCO)	3	—	—	—	—	—	1
Miami, FL (MIA)	4	—	—	—	2	2	1
Dallas, TX (DFW)	5	—	5	—	1	—	—
San Juan, Puerto Rico (SJU)	6	—	2	—	—	—	—
Seattle, WA (SEA)	7	—	—	—	1	—	—
San Diego, CA (SAN)	8	—	—	—	—	1	—
Los Angeles, CA (LAX)	9	1	1	—	—	2	—
San Jose, CA (SJC)	10	—	—	—	—	2	—
San Francisco, CA (SFO)	11	—	—	—	—	2	—

availability, and other operational requirements. The BTS data do not include this detailed information about the airline's operations; the data only provide the allocation of aircraft actually used by the airline on its routes from Boston. As a result, replicating the actual airline operations is impossible without the additional information needed to formulate the allocation problem the same way that the airline's operations research group formulated the problem.

Using the objective of minimizing direct operating cost of the fleet for a typical day of operations, an allocation of the airline's existing aircraft can be computed in a manner consistent with the simple three-route problem. To assess improvement of introducing a new aircraft, the typical daily operating cost of the existing fleet is needed. An integer programming approach provides a baseline allocation for the 11-route problem using only the six existing aircraft types. The demand from the Boston hub to the outlying airport follows the one-way demand idea. Each existing aircraft makes no more that two trips per day, and each trip is a round trip. Table 9 presents the baseline allocation of existing aircraft, and the predicted DOC for the entire fleet for one day of operations is $710,408.

Because of the assumptions and the DOC estimation method used in this study, the baseline allocation in Table 9 does not use the B767-200 aircraft, even though the BTS [34] data report that the actual airline used these aircraft to serve routes from Boston. Similarly, the number of flights listed in Table 9 is likely smaller than that associated with a typical day of the actual airline, because scheduling of flights throughout the day is not considered. The differences between the actual allocation and those in Table 9, however, do not reduce the validity of the methodology presented in this study. The integer programming problem can incorporate additional considerations for fleet allocation without changing the decomposition strategy. The approach of minimizing daily fleet DOC is consistent when allocating the existing fleet and the fleet that includes the yet-to-be-designed aircraft type X, and so the change in DOC between the two fleet allocation results should indicate the type of fleet-level improvements possible when introducing the new aircraft.

From here, the next step is to solve the allocation problem that includes the yet-to-be-designed aircraft. This problem has 77 allocation variables and four aircraft design variables; of these, 78 are integer and three are continuous. The sequential decomposition method solves this MINLP problem, and so the flowchart in Fig. 3 remains valid for this problem as well.

A difference from the three-route problem presented earlier is that FLOPS is used to size the type X aircraft and generate the DOC coefficients instead of the MATLAB aircraft sizing code. FLOPS offers a variety of optimization algorithms to assist with aircraft sizing: Davidon–Fletcher–Powell (DFP) algorithm, Broyden–Fletcher–Goldfarb–Shano (BFGS) algorithm, conjugate gradient (Polak–Ribiere) algorithm, steepest descent algorithm, and the Kreisselmeier–Steinhauser (KS) function with DFP algorithm [33]. For the nonlinear constrained problem, these methods use a penalty function approach to enforce the constraints. All of these appear to provide valid results for an example aircraft study, but most of the approaches have a number of user-input penalty function parameters that rely on trial and error to determine values that perform consistently for a specific problem. The KS function with DFP algorithm requires the fewest adjustments, and so this became the optimization algorithm used to solve the aircraft sizing subproblem within FLOPS.

The modified golden section search needed nine system-level iterations to solve the 11-route problem, which required about 9 min of computer time on a shared compute server with varying loads. Table 10 reports the optimal allocation obtained. Based upon several unsuccessful attempts, solving this as a monolithic MINLP problem is not practical due to the computational expense.

The allocation of the existing aircraft has changed from the baseline allocation. Now, the airline does not use aircraft type A and C, while the airline allocates the new aircraft type X to 10 trips. The optimal fleet DOC with the newly designed type X aircraft is $627,740, a reduction of about 12% from the baseline allocation. The optimal aircraft type X design variables had the following values: $AR_X = 9.5$, $(W/S)_X = 136.4$ lb/ft^2, $(T/W)_X = 0.3$, and passenger capacity $y_X = 224$. The passenger capacity suggests a moderately large, single-aisle aircraft. As with the three-route problem, the type X aircraft has a lower DOC per seat-mile than the existing aircraft. The design range for aircraft X matches the longest range in the given route structure (2350 n miles), which tailors the new aircraft for this 11-route operations, whereas several of the existing aircraft have design ranges greatly exceeding the longest range route. The

Table 10 Eleven-route problem sequential decomposition optimal allocation

BOS to:	Route no.	a/c A A300-600	a/c B B757-200	a/c C B767-200	a/c D B737-800	a/c E DC9 S80	a/c F Fokker 100	a/c X
JFK	1	——	——	——	——	——	1	——
ORD	2	——	4	——	2	——	——	——
MCO	3	——	——	——	——	——	1	——
MIA	4	——	——	——	——	——	——	3
DFW	5	——	——	——	1	——	——	4
SJU	6	——	2	——	——	——	——	——
SEA	7	——	——	——	1	——	——	——
SAN	8	——	——	——	——	1	——	——
LAX	9	——	——	——	——	——	——	2
SJC	10	——	——	——	——	——	——	1
SFO	11	——	——	——	——	2	——	——

DOC objective function, as before, does not account for costs of retiring existing aircraft nor does it directly account for fixed costs associated with acquisition of a new aircraft type.

The resulting aircraft description from FLOPS indicates a high aspect ratio for this 11-route problem, whereas the MATLAB sizing routine favors a low aspect ratio ($AR_X = 6.0$) for the previous three-route problem. This difference is due to the simplistic and less accurate contribution of the aspect ratio to the empty weight of the aircraft in the MATLAB routine, whereas FLOPS uses more detailed equations and approximations to compute the empty weight of the aircraft. The higher aspect ratio properly leads to a more cost-effective aircraft.

C. Thirty-One-Route Problem

Having shown that the decomposition approach can solve a larger problem than the three-route example, an even larger problem is attempted. Again, a major U.S. airline provides the basis for the problem. Data from the BTS database [34] describe this airline's domestic operations from Memphis, Tennessee, one of its main hubs, for the month of July 2003; this includes regularly scheduled routes and equipment used. The number of passengers carried on each route allows calculation of a daily demand value. Figure 6 displays the 31 routes served, and Table 11 presents the daily demand and route distance between Memphis and the 31 destination cities.

As with the 11-route problem, the problem deviates from the actual airline operations recorded in BTS. Here, Memphis now serves as the sole hub for the airline, and the daily demand numbers represent the one-way demand on round trips between Memphis and the 31 destinations. The number of existing aircraft owned by the airline is calculated from BTS data using the limit that each aircraft can only make two trips per day. From this calculation, the airline owns one aircraft type A, B757-300 with a passenger capacity of 224; seven aircraft type B, A320 with a passenger capacity of 148; two aircraft type C, B757-200 with a passenger capacity of 180; eight aircraft type D, A319 with a passenger capacity of 124; three aircraft type E, DC-9-50 with a passenger capacity of 125; sixteen aircraft type F, DC-9-30 with a passenger capacity of 100; and one aircraft type G, DC-9-10 with a passenger capacity of 78. FLOPS again sized these existing aircraft using available information about the aircraft. The design range of each aircraft indicates whether or not a given aircraft is able to fly the required routes and allows FLOPS to compute the performance coefficients for each aircraft on each economic mission.

As before, an integer programming problem solution provides a baseline allocation of the seven existing aircraft types (Table 12); the fleet DOC for this baseline allocation is \$708,336/day. This baseline solution uses all aircraft types, but does not reflect the allocation of the actual airline as reported in the BTS data. As for the 11-route problem, the simplification of time scheduling and the assumption of only round-trip flights are responsible for this difference in allocation.

The allocation of the existing fleet for the 31-route problem has a daily DOC that is slightly smaller (\$708,336) than that of the existing fleet allocation for the 11-route problem (\$710,408). Even though the 11-route problem is a smaller problem (fewer routes and aircraft types, and fewer variables and constraints) than the 31-route problem, the two existing fleet allocation problems employ different aircraft (with differing DOC values) and have different passenger demand. The 31-route problem has a demand of 4,700,885 passenger-miles, whereas the 11-route problem has a larger demand of 6,130,264 passenger-miles, and so the higher cost of the 11-route problem is believable.

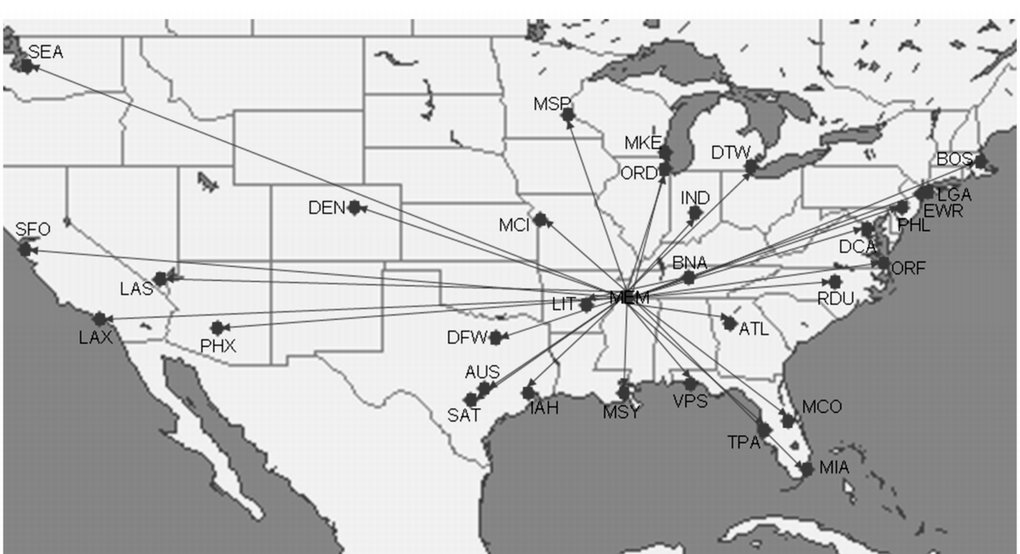

Fig. 6 Thirty-one-route allocation problem.

Table 11 Demand for each of 31 routes and distance from hub

Memphis (MEM) to:	Route no.	Daily demand, pax	Distance, n mile
Little Rock, AR (LIT)	1	99	113
Nashville, TN (BNA)	2	80	174
Atlanta, GA (ATL)	3	51	289
New Orleans, LA (MSY)	4	184	303
Valparaiso, FL (VPS)	5	263	324
Indianapolis, IN (IND)	6	169	331
Kansas City, MO (MCI)	7	158	342
Dallas/Ft. Worth, TX (DFW)	8	135	375
Houston, TX (IAH)	9	284	407
Chicago, IL (ORD)	10	184	427
Milwaukee, WI (MKE)	11	92	484
Austin, TX (AUS)	12	132	486
Detroit, MI (DTW)	13	754	531
San Antonio, TX (SAT)	14	150	543
Raleigh/Durham, NC (RDU)	15	238	550
Tampa, FL (TPA)	16	264	570
Orlando, FL (MCO)	17	365	594
Minneapolis, MN (MSP)	18	749	609
Washington, DC (DCA)	19	234	622
Norfolk, VA (ORF)	20	50	680
Miami, FL (MIA)	21	101	747
Denver, CO (DEN)	22	124	758
Philadelphia, PA (PHL)	23	175	760
Newark, NJ (EWR)	24	221	823
New York, NY (LGA)	25	258	837
Boston, MA (BOS)	26	105	991
Phoenix, AZ (PHX)	27	112	1098
Las Vegas, NV (LAS)	28	129	1231
Los Angeles, CA (LAX)	29	506	1407
San Francisco, CA (SFO)	30	134	1570
Seattle, WA (SEA)	31	266	1626

With these data at hand, the allocation of the problem including the yet-to-be-designed aircraft type X is investigated. Assigning a design range of 1626 n miles for the new aircraft ensures that the new aircraft can service every route in the 31-route structure, even though this range is much shorter than most commercial aircraft. As a result, the new aircraft will be tailored to this airline's operations and have a lower DOC per seat-mile than the existing aircraft. This problem has 248 allocation variables and four aircraft design variables; of these, 249 are integer and three are continuous variables. The sequential decomposition of Fig. 3 is used to solve this problem. Table 13 reports the optimal aircraft allocation obtained.

The airline uses all types of existing aircraft and the newly designed aircraft in this allocation, however, the allocation of the existing aircraft has changed. For instance, the number of trips flown by aircraft type F reduces from 23 to three. The optimal fleet DOC is \$617,742/day, a reduction of about 13% from the baseline allocation. The optimal aircraft type X design variables had the following values: $AR_X = 9.5$; $(W/S)_X = 135.2\ \mathrm{lb/ft^2}$, $(T/W)_X = 0.31$, and passenger capacity $y_X = 250$. The passenger capacity here is at its upper bound. The modified golden section search used nine system-level iterations to solve the problem. The time required to solve this problem was about 24.3 min on a shared computer server with varying loads. This increase in time over the 11-route problem reflects the increasing size of the allocation problem.

V. Conclusions

This work examined one type of design problem encountered in a system-of-systems context: that of allocating existing and yet-to-be-designed systems to provide a capability. The intent of the problem statements is to foster discussion about approaching allocation of variable resources using optimization methods in the context of system-of-systems. With some assumptions and simplifications, an optimization approach could solve the MINLP formulation of a simple airline allocation problem with a yet-to-be-designed aircraft. The solutions obtained for this extended allocation problem always

Table 12 Allocation of existing fleet for 31-route problem

MEM to:	Route no.	a/c A B757-300	a/c B A320-100	a/c C B757-200	a/c D A319	a/c E DC-9-50	a/c F DC-9-30	a/c G DC-9-10
LIT	1	—	—	—	—	—	1	—
BNA	2	—	—	—	—	—	1	—
ATL	3	—	—	—	—	—	—	1
MSY	4	—	—	—	—	—	2	—
VPS	5	—	1	—	—	1	—	—
IND	6	—	—	—	—	—	2	—
MCI	7	—	—	—	—	—	1	1
DFW	8	—	1	—	—	—	—	—
IAH	9	—	—	—	—	—	3	—
ORD	10	—	—	—	—	—	2	—
MKE	11	—	—	—	—	—	1	—
AUS	12	—	1	—	—	—	—	—
DTW	13	—	2	—	3	—	1	—
SAT	14	—	—	1	—	—	—	—
RDU	15	—	—	—	2	—	—	—
TPA	16	—	—	—	—	—	3	—
MCO	17	—	—	—	3	—	—	—
MSP	18	—	—	1	—	3	2	—
DCA	19	—	1	—	—	—	1	—
ORF	20	—	—	—	—	—	2	—
MIA	21	—	—	—	1	—	—	—
DEN	22	—	—	—	1	—	—	—
PHL	23	—	—	1	—	—	—	—
EWR	24	1	—	—	—	—	—	—
LGA	25	—	—	—	1	—	1	—
BOS	26	—	—	—	1	—	—	—
PHX	27	—	—	—	1	—	—	—
LAS	28	—	1	—	—	—	—	—
LAX	29	1	2	—	—	—	—	—
SFO	30	—	1	—	—	—	—	—
SEA	31	—	1	—	1	—	—	—

Table 13 Thirty-one-route problem sequential decomposition optimal solution

MEM to:	Route no.	a/c A B757-300	a/c B A320-100	a/c C B757-200	a/c D A319	a/c E DC-9-50	a/c F DC-9-30	a/c G DC-9-10	a/c X
LIT	1	——	——	——	——	——	1	——	——
BNA	2	——	——	——	——	——	1	——	——
ATL	3	——	——	——	——	——	——	1	——
MSY	4	1	——	——	——	——	——	——	——
VPS	5	——	1	——	——	1	——	——	——
IND	6	——	——	1	——	——	——	——	——
MCI	7	——	——	1	——	——	——	——	——
DFW	8	——	1	——	——	——	——	——	——
IAH	9	——	2	——	——	——	——	——	——
ORD	10	1	——	——	——	——	——	——	——
MKE	11	——	——	——	——	——	1	——	——
AUS	12	——	1	——	——	——	——	——	——
DTW	13	——	1	——	——	1	——	——	2
SAT	14	——	——	1	——	——	——	——	——
RDU	15	——	——	——	——	——	——	——	1
TPA	16	——	1	——	——	1	——	——	——
MCO	17	——	——	——	——	1	——	——	1
MSP	18	——	——	——	——	——	——	——	3
DCA	19	——	——	——	——	——	——	——	1
ORF	20	——	——	——	——	——	——	1	——
MIA	21	——	——	——	——	1	——	——	——
DEN	22	——	——	——	——	1	——	——	——
PHL	23	——	——	1	——	——	——	——	——
EWR	24	1	——	——	——	——	——	——	1
LGA	25	——	1	——	1	—	——	——	——
BOS	26	——	——	——	1	——	——	——	——
PHX	27	——	——	——	1	——	——	——	——
LAS	28	——	1	——	——	——	——	——	——
LAX	29	——	1	——	1	——	——	——	1
SFO	30	——	1	——	——	——	——	——	——
SEA	31	——	1	——	1	——	——	——	——

resulted in a direct operating cost per day lower than the allocation of only the existing aircraft. However, using a nonlinear branch-and-bound algorithm incurred significant computational cost and did not always find the appropriate solution for the simplest version of the airline problem.

A decomposition analogous to those used in multidisciplinary design optimization was sought for the mixed-integer, nonlinear problem statement. The decomposition approach separates the problem into discipline-specific subproblems of aircraft sizing and resource allocation. For the problem described here, the decomposition was sequential; for a small change in the problem, this might not be the case. The decomposition separates the aircraft sizing, an NLP problem, from the resource allocation, an IP problem. In doing so, the approach can use good integer programming algorithms to solve the resource allocation subproblem and good nonlinear programming algorithms to solve the aircraft sizing subproblem. In this investigation, the optimization routines available within FLOPS solve the nonlinear aircraft sizing subproblem, and GAMS (using CPLEX) solves the integer allocation subproblem.

By decomposing the problem, the optimization approach can still obtain a solution when increasing problem size. In the studies presented here, increasing the size of the problem enlarged the size of the resource allocation subproblem while keeping the size of the aircraft sizing subproblem constant. The computational effort for the aircraft sizing problem increases slightly because more economic missions have to be evaluated. The branch-and-bound algorithm was unable to find solutions for the 11-route problem and the 31-route problem.

The decomposition allows for interchange of problem specific tools when solving the subproblems. Following the example in this study, any other aircraft sizing software can be used with minimal compatibility issues. As other tools become available, the model can easily include them. Similarly, the complexity of the aircraft sizing problem can also increase without requiring a change in the decomposition.

In traditional aircraft design, as with the design of many systems (like ground vehicles, satellites, etc.), a set of design requirements guides the design with the intent that the new aircraft will provide better capabilities for the operator by meeting these requirements. However, this process does not directly account for the impact of the new aircraft on the operator. In the examples presented in this paper, an airline is the operator. The airline uses many aircraft for its purposes, and the DOC of the entire fleet, not only one aircraft, is a measure of the cost of its operations. The DOC for a fleet of aircraft is a function of the DOC of the individual aircraft and of the allocation of those aircraft. And so, when choosing the values of the aircraft design variables, it is not enough to evaluate the cost of operating the new aircraft, but also the impact that it may have on the entire fleet and its allocation. It was the scope of this study to investigate this relationship and to identify a way of designing aircraft that positively affects not only aircraft design but also airline operations. Here, including data describing the behavior of a potential addition to the fleet, and tailoring/optimizing the design variables of that aircraft to best fit the operation structure of the airline, helps improve both industries. Fleet allocation has a large impact not only on the operating costs of an airline but should also directly impact the design of the aircraft themselves. The decomposition approach to allow allocation of existing and yet-to-be-designed systems appears to meet this aim of designing a new system to improve operations rather than to improve the individual system against a set of requirements. To the authors' best knowledge, this work represents the first attempt to cast this system-of-systems inspired problem as an MINLP problem and then solve it via a decomposition strategy.

In recent years, the government, particularly the U.S. military, has been asking for capabilities rather than specific vehicles designed to an exacting set of requirements. For instance, in the U.S. Coast Guard Integrated Deepwater System, the Coast Guard did not specify the mix of assets it wanted the contractors to provide, but it gave them the freedom of deciding the design characteristics and specifications of the new assets, and the mix of new and existing systems that would allow the Coast Guard to perform its mandated missions. The approach to formulate the allocation of variable resources problem for the airline and solve it via decomposition could have application to problems like the Deepwater missions.

This effort is not an all-encompassing investigation for solving system-of-systems problems that have features of allocating variable resources. The problem presented here is a simplification of a real-life scenario. Although the decomposition approach found solutions to the 11- and 31-route airline problems when the monolithic MINLP approach could not, the decomposition strategy may have upper bounds on problem size, where the allocation and aircraft sizing problems become so large themselves that the decomposition strategy becomes computationally prohibitive.

The set of variables employed in this work illustrates the benefit of viewing the allocation of a yet-to-be-designed aircraft along with existing aircraft as a decomposable MINLP problem, however, both the airline allocation subproblem and the aircraft sizing subproblem could incorporate many additional variables. This could allow for more detail in the aircraft sizing and for more definition of the airline allocation problem. Replacing the airline allocation problem with an airline scheduling problem would use the same decomposition, but the scheduling problem (in which individual aircraft need to be tracked) is often posed as a binary integer programming problem. Another problem formulation including the design range of the new aircraft as a top-level variable would make the top-level problem a mixed-integer problem, albeit a very small one. The golden section search would no longer be appropriate at the top level. Route design and crew scheduling present additional items of interest that might warrant inclusion into this type of problem; in these cases, a multidomain decomposition might be appropriate.

For the example of using this approach for airline problems, the direct operating cost is not the only cost an airline has to consider. When an airline allocates and uses its aircraft, it must consider many factors including scheduling of flights during the day, the location of its maintenance facilities, the maintenance schedule for each aircraft, crew availability, cost of maintenance, cost of facilities, and other operational requirements. Including some or all of these in the allocation problem formulation will increase the fidelity of the results.

The work described in this paper minimized direct operating cost as a surrogate for maximizing profit. There are also costs related to the acquisition of a new asset that the results do not capture. No provisions are made for the financing cost of acquiring a new aircraft. In the scenarios presented in the study, the solution often did not allocate a number of the existing aircraft after introduction of the new aircraft. This model does not capture the airline expenditures associated with parking an aircraft or retiring it. The cost prediction model does, however, include depreciation and interest, but decisions regarding the useful life of an aircraft vary significantly from operator to operator.

The simplifications used in this paper allowed an initial investigation of a system-of-systems-inspired problem in which the design of a new system (here, aircraft sizing) proceeded in collaboration with the design of how that system will be operated (here, allocation of the new aircraft along with existing aircraft). Extensions of this work could include addressing uncertainty in the problem (e.g., the fluctuation of passenger demand), converting the airplane allocation problem to an airplane scheduling problem (e.g., allow for timing of flights, including consideration of connecting flights), including multiple competing airlines (e.g., the new aircraft needs to support operations of two different route structures), addressing airline fleet evolution over time (e.g., new aircraft do not instantly appear), and incorporating the route network design in collaboration with the new aircraft sizing and allocation (e.g., a new aircraft could enable new routes in the hub-and-spoke and/or allow point-to-point operations).

The simple problems and investigations described in this paper suggest a promising decomposition approach for one type of system-of-systems problem. It is hoped that these investigations will lead to further discussion and efforts to build the tool set needed for designing systems-of-systems.

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